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# A natural solution of $\mu$ from the hidden sector

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## Abstract

The  $\mu$  parameter is calculated in supergravity models possessing the  $U(1)_A \times U(1)_R$  symmetry. In one natural model without a free mass parameter below the Planck scale, the symmetry breaking scale is identified as the hidden sector squark condensation scale.

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## I. INTRODUCTION

The  $\mu$  problem [1] is one of the mass hierarchy problem reintroduced in supersymmetric models toward a gauge hierarchy solution. In the literature, many mechanisms to resolve this unexpected  $\mu$  were proposed [1–4]. Among these solutions, the existence of an underlying symmetry to forbid a large  $\mu$  term is most compelling [5]. The so-called Giudice-Masiero mechanism [2] must also assume a symmetry, otherwise the absence of  $M_{Pl}H_1H_2$  in the superpotential is not guaranteed. A  $U(1)$  global symmetry or  $R$  symmetry is enough to forbid the  $\mu$  term at tree level in the superpotential [6].

In supergravity, the Kähler potential  $K$  and the superpotential  $W$  have the symmetry

$$G = K + \log |W|^2 = \text{invariant.} \quad (1)$$

Therefore, even if the  $\mu$  term is forbidden in  $W$  by a symmetry (say, by the  $R$  symmetry with  $R = 0$  for the  $H_1H_2$  operator), one can write it in  $K$  if the symmetry allows it. For example, one may consider  $[\epsilon H_1H_2/M_P^2 + \text{h.c.} + \log |W'|^2]$  which is the same as  $\log |W'|^2$  through the relation (1) where

$$W' = We^{\epsilon H_1H_2/M_P^2}$$

where  $M_P = M_{Pl}/\sqrt{8\pi} = 2.44 \times 10^{18}$  GeV. Thus the supergravity introduces nonrenormalizable terms suppressed by  $M_P$  and the  $\mu$  term is generated at the level  $\langle W \rangle /M_P^2 \sim m_{3/2}$ . Is this enough to state that supergravity has a natural scale for  $\mu$  even without a symmetry principle?

If we require that the theory dictates no superpotential at some basis, then any interaction must come from the Kähler potential,

$$G = K_0 + \frac{1}{M_P^3} (W_0 + \bar{W}_0) + \log |1|^2. \quad (2)$$

By the transformation (1)  $G = K_0 + \log |e^{W_0/M_P^3}|^2$ , we obtain a superpotential including definite nonrenormalizable terms,

$$W = \Lambda^3 e^{W_0/M_P^3} \quad (3)$$

where  $\Lambda$  is a mass parameter. Requiring that  $W$  contains the known Yukawa interactions,  $\Lambda$  must be of order  $M_P$ , implying  $\langle F \rangle \simeq M_P^2$  and  $m_{3/2} \sim M_P$ . Writing the Kähler potential as  $G$  of Eq. (2),  $G$  must contain  $O(M_P)H_1H_2 + \text{h.c.}$ , if no global symmetry is imposed in  $G$ . Here the mass parameter must be of order  $M_P$ . Then  $W$  contains the  $\mu$  term of order  $M_P$ . From this example, we note that the TeV scale  $\mu$  term is not a generic feature from Kähler potential in supergravity.

Therefore, there is a need to define  $W$  more clearly. In this paper, we *define the superpotential  $W$  as the maximum obtained from the transformation (1)*, i.e. when there is no piece left in the Kähler potential which can be transformable to  $W$ . This is the reason why we must include all possible nonrenormalizable terms in  $W$ . In this basis, we can effectively apply the nonrenormalization theorem in  $W$ . In the following, we respect the symmetry both in the superpotential and Kähler potential. The Kähler potential may be split into  $K_1 + K_2$  where  $K_1$  is nonholomorphic and  $K_2$  is holomorphic in the sense  $K_2 = W_2 + \text{h.c.}$  Then we take  $K_1$  as the Kähler potential and  $K_2$  must be included in the superpotential. If  $K_2$  were not respecting a global symmetry of the original superpotential, then our final superpotential would not respect the global symmetry. Therefore, the symmetry principle we impose on the superpotential must apply also to the Kähler potential. Under this symmetry principle, various possibilities of generating the electroweak scale  $\mu$  were considered before [5].

## II. THE $U(1)_A \times U(1)_R$ SYMMETRY

Two most popular global symmetries in supersymmetric models are the Peccei-Quinn symmetry  $U(1)_A$  and  $U(1)_R$  symmetry. Of course, the symmetries are respected by the Kähler potential, and there is no piece left in  $K$  which can be transformed to  $W$ . In this spirit, we must include all nonrenormalizable terms in  $W$ .

We impose the symmetry  $U(1)_A \times U(1)_R$ . The relevant fields for our purpose are listed in Table 1 with their quantum numbers.

Table 1. The  $A$  and  $R$  quantum numbers of superfields.

	$H_1$	$H_2$	$S_1$	$S_2$	$S_3$	$S_4$	$Z$
$A$	-1	-1	1	-1	2	-2	0
$R$	0	0	1	-1	2	0	2

The most general superpotential consistent with the  $U(1)_A \times U(1)_R$  symmetry, for  $d \leq 4$ , is

$$W = f_0 H_1 H_2 S_3 + f_1 Z (S_1 S_2 - F^2) + M S_3 S_4 + f_2 S_1^2 S_4 + \frac{f_3}{M_P} H_1 H_2 S_1^2 \quad (4)$$

where  $F$  is a mass parameter at  $10^{12-13}$  GeV scale and  $M$  is of order  $M_P$ . Thus, when  $S_1$  obtains a vacuum expectation value of order  $F$ , we obtain a reasonable  $\mu$

$$\mu = \frac{f_3}{M_P} F^2. \quad (5)$$

In this scheme, there is no contribution to  $\mu$  from the Kähler potential. To have a contribution to  $\mu$  *a la* Giudice and Masiero [2], we must introduce a singlet field  $S_5$  with  $A = -2, R = 0$ , so that the Kähler potential includes  $S_5^* H_1 H_2 + \text{h.c.}$  Then we obtain

$$\int d^2\bar{\theta} \int d^2\theta \frac{1}{M_P} S_5^* H_1 H_2 = \int d^2\theta \frac{F_{S_5^*}}{M_P} H_1 H_2 \quad (6)$$

from which  $\mu$  term is interpreted as  $F_{S_5^*}/M_P$ . If the F-term of  $S_5$  is nonvanishing and is of order  $10^{11}$  GeV, then we obtain a correct order of  $\mu$ . This necessarily assumes a supersymmetry breaking mechanism, which is contrasted to  $\mu$  arising from VEV of a scalar field given in Eq. (5).

### III. A NATURAL MODEL FOR $\mu$

In the above example, the intermediate scale parameter  $F$  is inserted by hand. Also, if the Giudice-Masiero mechanism is to be introduced, a specific form of the supersymmetry breaking at the intermediate scale must be assumed. Therefore, we must include the intermediate scale physics. Moreover, it is natural that if any gauge singlet is introduced, the

mass parameter accompanying the singlet is of order of the Planck scale. Thus it is better if this scale  $F$  of Eq. (4) derives from the h-sector confining force, rather than putting it by hand in the superpotential. Along this line, we introduce an h-sector gauge group as  $SU(N)_h$ . In Ref. [3] this idea has been proposed to generate a  $\mu$  term from the hidden sector squark (h-squark) condensation through the nonrenormalizable term,

$$\frac{1}{M_P} H_1 H_2 \bar{Q}_1 Q_2.$$

But, in Ref. [3], it was not given how  $\bar{Q}_1 Q_2$  develops a vacuum expectation value.

We proceed to discuss to generate  $\langle \bar{Q}_1 Q_2 \rangle$  from the intermediate scale physics. Let us consider the fields given in Table 2 with the  $U(1)_A \times U(1)_R$  symmetry. We introduce two chiral h-quarks  $Q_2, Q_4$  and two chiral anti-h-quarks  $\bar{Q}_1, \bar{Q}_3$ . For the h-sector  $SU(N)_h$  gauge group, these transform as  $N$  and  $N^*$ , respectively.

Table 2. The  $A$  and  $R$  quantum numbers with h-quarks  $Q_i$ .

	$H_1$	$H_2$	$\bar{Q}_1$	$Q_2$	$\bar{Q}_3$	$Q_4$	$S$	$S'$
$A$	-1	-1	1	1	1	1	-2	2
$R$	0	0	1	1	-1	-1	0	2

The  $d \leq 4$  superpotential consistent with the symmetry is

$$W = MSS' + H_1 H_2 S' + \bar{Q}_1 Q_2 S + \frac{1}{M_P} H_1 H_2 \bar{Q}_1 Q_2 \quad (7)$$

where we suppressed the couplings of order 1. Due to the symmetry, there cannot appear  $\bar{Q}_1 Q_2, \bar{Q}_3 Q_4, \bar{Q}_1 Q_4$ , and  $Q_2 \bar{Q}_3$  terms at the Planck scale. From the symmetry, we expect the nonrenormalizable term given in Eq. (7), which is the result of supergravity effects. But below the h-sector scale, even without the nonrenormalizable term given in Eq. (7), we may consider the effect of the  $S, S'$  sfermion exchange diagram, and the suppression factor is of order  $M_P$  since  $M$  in Eq. (7) is of order  $M_P$  from the naturalness argument. On the other hand, even without the nonrenormalizable term in Eq. (7),  $\partial W / \partial S = 0$  gives  $S' = -\bar{Q}_1 Q_2 / M$

which, after inserted in the  $H_1 H_2 S'$  term of (7), gives the desired nonrenormalizable term below the h-sector scale. In any case, below the h-sector scale we consider Eq. (7). With  $\langle \bar{Q}_1 Q_2 \rangle \sim \Lambda_h^2$ , we obtain  $\mu$  of order the electroweak scale [3].

The singlet fields  $S$  and  $S'$  are removed at the Planck scale. At low energy there remain  $H_1, H_2, \bar{Q}_1, Q_2, \bar{Q}_3$ , and  $Q_4$ . The h-gluinos can couple to the h-quarks through the h-sector strong interactions to give

$$\int d^2\theta W^\alpha W_\alpha \left( \frac{1}{4} + f(W^\alpha W_\alpha, \text{Det } \bar{Q}Q) \right)$$

where the first factor comes from the h-gauge sector kinetic energy term and the second factor is the result of the h-sector dynamics and is a function of two arguments respecting the global symmetry below the h-sector scale. This global symmetry is  $SU(N_f)_1 \times SU(N_f)_2 \times U(1)_B \times U(1)_C \times U(1)_{\tilde{R}}$  where  $U(1)_C$  is anomalous and  $U(1)_{\tilde{R}}$  is anomaly free. These quantum numbers of the h-sector fields are given in Table 3.  $C$  and  $\tilde{R}$  are linear combinations of  $A$  and  $R$ . But for the study of h-sector dynamics,  $C$  and  $\tilde{R}$  are more convenient. The h-sector scale  $\Lambda_h$  has the nontrivial  $C$  transformation property to match anomaly [9]. In the table, the composite meson field  $T = \bar{Q}Q$  is also shown.

Table 3. The  $SU(N_f)_1 \times SU(N_f)_2$ ,  $C$  and  $\tilde{R}$  quantum numbers.

	$SU(N_f)_1$	$SU(N_f)_2$	$B$	$C$	$\tilde{R}$
$W^\alpha$	1	1	0	0	1
$Q_i$	$N_f$	1	1	1	$-(N_c - N_f)/N_f$
$\bar{Q}_i$	1	$\bar{N}_f$	-1	1	$-(N_c - N_f)/N_f$
$T$	$N_f$	$\bar{N}_f$	0	2	$-2(N_c - N_f)/N_f$
$\Lambda_h^{3N_c - N_f}$	1	1	0	$2N_f$	0

For  $N_c = 2$  and  $N_f = 1$ , the 't Hooft instanton interaction with  $2N_c$  gluino lines and  $2N_f$  quark lines is derived from

$$S_{ins} = \int d^2\theta (W^\alpha W_\alpha)^2 \left( \frac{\text{Det } \bar{Q}Q}{\Lambda_h^5} \right).$$

Consistently with the global symmetry of Table 3, we can write many terms of the form

$$\int d^2\theta (W^\alpha W_\alpha)^n \left( \frac{\Lambda_h^{3N_c-N_f}}{\text{Det} \bar{Q} Q} \right)^{\frac{n-1}{N_f-N_c}}, \quad (8)$$

where  $n$  is a nonnegative integer. However, the information on the anomaly matching fixes the relative coefficient to give [9]

$$\int d^2\theta S \left[ \log \frac{S^{N_c-N_f} \text{Det} T}{c^{1/3} \Lambda_h^{3N_c-N_f}} - (N_c - N_f) \right], \quad (9)$$

where  $S = W_\alpha W^\alpha$  and  $T_{ij} = \bar{Q}_i Q_j$ , and  $c$  is a number of order 1. The symmetry dictates the form of the effective interaction to the above form, Eq. (8), which coincides with the 't Hooft interaction for  $N_c = 2, N_f = 1$  and  $n = 2$ . Even though these cannot be generated perturbatively, but can be generated nonperturbatively since these terms respect the global symmetries [10,11]. The relevant scale for the nonperturbative generation of the above terms is the h-sector scale  $\Lambda_h$ .

For illustration we take  $N_c = 3$  and  $N_f = 2$  corresponding to two flavors of Table 2. Below the h-sector scale, let us represent

$$S \equiv W^\alpha W_\alpha = m^2 Z, \quad T_{ij} \equiv \bar{Q}_i Q_j = m' \Phi_{ij} \quad (10)$$

where  $Z$  and  $\Phi_{ij}$  are the effective chiral superfields, and  $m$  and  $m'$  are at the h-sector scale. Let us apply  $SU(N_f)_1 \times SU(N_f)_2$  transformation so that the matrix  $\Phi$  is diagonal  $\Phi_{ij} = \Phi_i \delta_{ij}$ . Then the relevant superpotential below the h-sector scale is given by

$$W_{eff} = -m^2 Z + m^2 Z \log \frac{m^2 m'^2 Z \Phi_1 \Phi_2}{c^{1/3} \Lambda_h^7}. \quad (11)$$

Minimization of  $W_{eff}$  gives

$$Z \Phi_1 \Phi_2 = \frac{c^{1/3} \Lambda_h^7}{m^2 m'^2}, \quad Z = 0, \quad \Phi_1 = \Phi_2 = \infty. \quad (12)$$

This runaway solution is a typical feature of the massless supersymmetric QCD. Therefore, in the massless theory, we may have only a cosmological interpretation for a nonzero  $Z$ .

If two quarks obtain masses of  $m_1$  and  $m_2$ , we add the following terms in the effective superpotential, Eq. (11),

$$- m_1 \bar{Q}_1 Q_2 - m_2 \bar{Q}_3 Q_4. \quad (13)$$

From Eq. (7), we note  $m_1 \simeq < S >$  and  $m_2 = 0$ . After the introduction of soft supersymmetry breaking terms at order  $m_{3/2}$ ,  $S$  develops a vacuum expectation value of order  $m_{3/2} \Lambda_h^2 / M^2$ , which implies  $m_1 \sim 10^{-10} m_{3/2}$ . The minimization of  $W_{eff}$  gives

$$\frac{m^2 Z}{\Phi_1} - m_1 m' = 0, \quad \frac{m^2 Z}{\Phi_2} - m_2 m' = 0 \quad (14)$$

in addition to the first equality of Eq. (12). These cannot be satisfied simultaneously. Supersymmetry is broken. Still  $\Phi_2$  runs away. We will cutoff  $\Phi_2$  at  $M_P$ , for which we try to introduce a nonvanishing  $m_2 \sim 10^{-6} m_1$  by hand. For nonzero  $m_2$ , we can satisfy Eq. (14), and determine

$$Z \sim 10^{-11} m_{3/2} \sim 1 \text{ eV}, \quad \Phi_1 \sim 10^{-1} \frac{\Lambda_h^2}{m'} \sim 10^{12} \text{ GeV}, \quad \Phi_2 \sim 10^5 \frac{\Lambda_h^2}{m'}. \quad (15)$$

Since  $\Phi_1 \sim 10^{12}$  GeV, we obtain a ball park  $\mu$ .

Since both  $U(1)_A$  and  $U(1)_R$  symmetries are broken by the vacuum expectation values of  $\Phi_1, \Phi_2$  and  $Z$ , there result two Goldstone bosons. One is the familiar invisible axion (or the very light axion) [12] and the other is an R-axion. The R-axion is the pseudo-Goldstone boson resulting from the breaking of the  $U(1)_R$  symmetry. The model given in Table 2 have the nonvanishing divergences for both the  $J_\mu^A$  and  $J_\mu^R$  currents,

$$\partial^\mu J_\mu^A = - \frac{4}{32\pi^2} F' \tilde{F}' \quad (16)$$

$$\partial^\mu J_\mu^R = \frac{2N_c}{32\pi^2} F' \tilde{F}' \quad (17)$$

where  $F' \tilde{F}'$  is the h-sector gluon anomaly,  $(1/2) \epsilon^{\mu\nu\rho\sigma} F'_{\mu\nu} F'_{\rho\sigma}$ . Even if the h-sector scale is  $\Lambda_h$ , the instanton potential is multiplied by a factor  $m_{\tilde{G}}^{N_c} m_1 m_2 / \Lambda_h^{N_c-1}$  where  $m_{\tilde{G}}$ ,  $m_1$ , and  $m_2$  are the masses of the h-gluino, fermionic partners  $\bar{Q}_1$  (and its partner  $Q_2$ ) and  $\bar{Q}_3$  (and its partner  $Q_4$ ), respectively.  $m_{\tilde{G}}$  is expected to be of the electroweak scale order. A nonzero  $m_2$  can occur from the nonrenormalizable terms in  $W$ , but these are too small to give Eq. (14). Possible terms in the Kähler potential are more suppressed. In Eq. (13), we added  $m_2 \sim 10^{-6} m_1$  which is small enough not to invalidate our symmetry argument.

Because  $H_1$  and  $H_2$  carry Peccei-Quinn quantum numbers, Eq. (16) has a QCD gluon anomaly if QCD quarks are included. There are two decay constants,  $\langle \Phi_1 \rangle$  and  $\langle \Phi_2 \rangle$ . Because the h-sector instanton potential is very shallow, the decay constant corresponding to the QCD potential is the smaller one,  $\langle \Phi_1 \rangle$  [13]. The resulting axion is the very light one [12] with the decay constant  $\sim \langle \Phi_1 \rangle$  [13], and its nature is of composite type [14] since the Peccei-Quinn symmetry is broken by the h-squark condensation.<sup>1</sup> The other pseudo-Goldstone boson is extremely light with decay constant  $\sim \langle \Phi_2 \rangle$ .

#### IV. CONCLUSION

In conclusion, we have emphasized that the solution of the  $\mu$  problem should have a root with the symmetry principle. Along this line, the  $U(1)_A \times U(1)_R$  symmetry is used to generate an electroweak scale  $\mu$  naturally from the dynamics of the hidden sector. This class of models has a potential to house the much needed quintessence since the symmetry we require may forbid the h-quark masses down to a needed level [13]. However, the example we presented here requires the introduction of a nonzero parameter  $m_2$ . A more satisfactory solution would be to determine it dynamically.

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<sup>1</sup> The present model is much simpler than the intricate composite model of Ref. [15].

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